

1084 a. Dokazati da je za sve prirodne brojeve $n \geq 0$: $3 | 5^n + 2^{n+1}$

Za $n = 1$

$$\begin{aligned}3 &| 5^1 + 2^{1+1} \\3 &| 5^1 + 2^2 \\3 &| 9\end{aligned}$$

Pretpostavka da za $n = k$, $3 | 5^k + 2^{k+1}$

Za $n = k+1$

$$\begin{aligned}3 &| 5^{k+1} + 2^{k+1+1} \\3 &| 5^{k+1} + 2^{k+2} \\3 &| 5 \cdot 5^k + 2 \cdot 2^{k+1}\end{aligned}$$

I način

$$\begin{aligned}3 &| 3 \cdot 5^k + 2 \cdot 5^k + 2 \cdot 2^{k+1} \\3 &| 3 \cdot 5^k + 2 \cdot (5^k + 2^{k+1}) \\3 &| 3 \cdot 5^k \text{ i } 3 | 2 \cdot (5^k + 2^{k+1}) \text{ (pretpostavka)}\end{aligned}$$

II način

$$\begin{aligned}3 &| 5 \cdot 5^k + 5 \cdot 2^{k+1} - 3 \cdot 2^{k+1} \\3 &| 5 \cdot (5^k + 2^{k+1}) - 3 \cdot 2^{k+1} \\3 &| 5 \cdot (5^k + 2^{k+1}) \text{ i } 3 | 3 \cdot 2^{k+1}\end{aligned}$$